# Semileptonic $B$ Meson Decays Into A Highly Excited Charmed Meson Doublet

Long-Fei Gan[*] and Ming-Qiu Huang

*Department of Physics, National University of Defense Technology, Hunan 410073, China*

(Dated: June 27, 2009)

We study the heavy quark effective theory prediction for semileptonic $B$ decays into an orbital excited $F$-wave charmed doublet, the $(2^+, 3^+)$ states $(D_2^{*\prime}, D_3)$, at the leading order of heavy quark expansion. The corresponding universal form factor is estimated by using the QCD sum rule method. The decay rates we predict are $\Gamma_{B\to D_2^{*\prime}\ell\overline{\nu}} = 1.85\times10^{-19}$GeV and $\Gamma_{B\to D_3\ell\overline{\nu}} = 1.78\times10^{-19}$GeV. The branching ratios are $\mathcal{B}(B\to D_2^{*\prime}\ell\overline{\nu}) = 4.6\times10^{-7}$ and $\mathcal{B}(B\to D_3\ell\overline{\nu}) = 4.4\times10^{-7}$, respectively.

PACS numbers: 13.20.He, 11.55.Hx, 12.38.Lg, 12.39.Hg



The semileptonic $B$ decay processes with charmed mesons in their final states have attracted attention in recent years for the important role that they played in the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, such as $|V_{cb}|$. They also provide a useful testing ground for the strong interaction as well as heavy quark symmetry. Theoretically, the heavy quark effective theory (HQET) [1] has been well developed and widely applied to these processes. In HQET, the heavy quark spin-flavor symmetry brings much convenience to understanding the heavy-light mesons and their decays. On the one hand, the heavy-light meson spectrum can be organized according to the parity $P$ and the total angular momentum $s_l$ of the light degrees of freedom. Coupling the spin of the light degrees of freedom $s_l$ with the spin of a heavy quark $s_Q = 1/2$ yields a doublet of meson states with a total spin $s = s_l \pm 1/2$. The spectra of the heavy-light mesons were given early in Ref. [2] by a relativistic quark model. The properties of the ground and low-lying states have been extensively studied using different approaches during the past few years [3, 4, 5, 6]. On the other hand, the weak transition matrix elements describing semileptonic $B$ decays into a charmed meson can be parametrized, respectively, by one universal Isgur-Wise function at the leading order of heavy quark expansion [7, 8]. This simplifies the theoretical calculation of these processes dramatically. Experimentally, some of the $S$-wave and $P$-wave charmed states have been observed so far. Their masses and quantum numbers have been well confirmed. In addition, besides the measurement of the $B\to D^{(*)}\ell\overline{\nu}$ decay which accounts for most of the inclusive semileptonic $B$ decay branching ratio, more and more subdominant decay channels have been measured with an increasing accuracy[9, 10, 11].

* lfgan@nudt.edu.cn

Semileptonic $B$ decays into low-lying charmed mesons were studied in the past [8, 12, 13, 14]. In our previous work, we have given the HQET predictions for the semileptonic $B$ decays into the $D$-wave charmed meson doublets [15]. We find that the branching ratios are of the same order as those of the CKM suppressed $b \to u\ell\nu$ processes. So we can expect that, with the increasing of the orbital angular momenta of final excited charmed mesons, the dynamical suppression becomes severer than the CKM suppression. To make this clear, we study the HQET predictions for the semileptonic $B$ decays into the next heavy meson doublet, the $s_l^P = \frac{5}{2}^+$ doublet which comprises two mesons, $(D_2^{*'}, D_3)$, with $J^P = 2^+$ and $J^P = 3^+$.

The theoretical description of semileptonic decays involves the matrix elements of vector and axial vector currents ( $V^\mu = \bar{c}\gamma^\mu b$ and $A^\mu = \bar{c}\gamma^\mu\gamma_5 b$ ) between $B$ mesons and $D$ mesons, which can be parametrized into one universal form factor at the leading order of the heavy quark expansion by applying the trace formalism [16]. For the processes $B \to (D_2^{*'}, D_3)\ell\overline{\nu}$, these matrix elements turn out to be

$$\begin{aligned}\langle D_2^{*'}(v^{'},\varepsilon)|(V-A)^\mu|B(v)\rangle &= \sqrt{\frac{5}{3}}\sqrt{m_B m_{D_2}}\tau(y)\varepsilon^*_{\alpha\beta}v^\alpha[\frac{2(y^2-1)}{5}g^{\mu\beta} + v^\beta v^\mu - \frac{2y+3}{5}v^\beta v'^\mu \\ &+ i\frac{2(y-1)}{5}\epsilon^{\mu\lambda\beta\rho}v_\lambda v^{'}_\rho], \end{aligned} \tag{1}$$

$$\langle D_3(v^{'},\varepsilon)|(V-A)^\mu|B(v)\rangle = \sqrt{m_B m_{D_3^*}}\tau(y)\varepsilon^*_{\alpha\beta\lambda}v^\alpha v^\beta[g^{\mu\lambda}(y-1) - v^\lambda v'^\mu - i\epsilon^{\mu\lambda\rho\tau}v_\rho v^{'}_\tau], \tag{2}$$

where $y = v\cdot v^{'}$, and $\varepsilon^*_{\alpha\beta}$, $\varepsilon^*_{\alpha\beta\lambda}$ are the polarization tensors of these mesons. $\tau(y)$ is the Isgur-Wise function which can be estimated by using the QCD sum rule method [17].

For this purpose, some appropriate interpolating currents are needed to represent these states. Here we adopt the interpolating currents proposed in Ref. [3] based on the study of the Bethe-Salpeter equation for heavy mesons. For the $F$-wave meson doublet with $s_l^P = \frac{5}{2}^+$, the currents are given as follows:

$$J^{\dagger\alpha\beta}_{2,+,5/2} = i\sqrt{\frac{5}{6}}T^{\alpha\beta,\mu\nu}\overline{h}_v(D_{t\nu}D_{t\mu} - \frac{2}{5}D_{t\nu}\gamma_{t\mu}\not{D}_t)\not{D}_t q, \tag{3}$$

$$J^{\dagger\alpha\beta\lambda}_{3,+,5/2} = \frac{i}{\sqrt{2}}T^{\alpha\beta\lambda,\mu\nu\sigma}\overline{h}_v\gamma_5\gamma_{t\mu}D_{t\nu}D_{t\sigma}\not{D}_t q, \tag{4}$$

where $h_v$ is the generic velocity-dependent heavy quark effective field in HQET and $q$ denotes the light quark field; $D_{t\mu} = D_\mu - v_\mu(v\cdot D)$ is the transverse component of the covariant derivative with respect to the velocity of the meson, where $\gamma_{t\mu} = \gamma_\mu - \not{v}v_\mu$ is the transverse component of $\gamma_\mu$ with respect to the heavy quark velocity. The tensors $T^{\alpha\beta,\mu\nu}$ and $T^{\alpha\beta\lambda,\mu\nu\sigma}$ are used to symmetrize indices [3, 15]. These currents have nice properties: they have nonvanishing projection only to the corresponding states of the HQET in the $m_Q \to \infty$ limit, without mixing with states of the same quantum number but different $s_l$. Thus we can define one-particle-current couplings as follows:

$$\langle D_2^{*'}(v,\varepsilon)|J^{\alpha\beta}|0\rangle = f_2\sqrt{m_{D_2^{*'}}}\varepsilon^{*\alpha\beta}, \text{ for} J^P = 2^+; \tag{5}$$

$$\langle D_3(v,\varepsilon)|J^{\alpha\beta\lambda}|0\rangle = f_3\sqrt{m_{D_3}}\varepsilon^{*\alpha\beta\lambda},\ \text{for} J^P = 3^+. \tag{6}$$

The decay constants $f_2$ and $f_3$ are low-energy parameters which are determined by the dynamics of the light degrees of freedom. Hereafter, we denote them as $f_{+,\frac{5}{2}}$ in common.

The decay constants of heavy-light mesons can be estimated from two-point sum rules. Here we list them after the Borel transformation as follows. For the initial ground-state $B$ meson we consider, the sum rule for the correlator of two heavy-light currents of the ground-state is [13]

$$f^2_{-,\frac{1}{2}}e^{-2\bar{\Lambda}_{-,1/2}/T} = \frac{3}{16\pi^2}\int_0^{\omega_{c0}}\omega^2 e^{-\omega/T}d\omega - \frac{1}{2}\langle\bar{q}q\rangle(1-\frac{m_0^2}{4T^2}). \tag{7}$$

For the final $s_l^P = \frac{5}{2}^+$ doublet, the corresponding two-point sum rule is

$$\begin{aligned} f^2_{+,\frac{5}{2}}e^{-2\bar{\Lambda}_{+,5/2}/T} &= \frac{1}{5\times 2^9\pi^2}\int_0^{\omega_{c1}}\omega^8 e^{-\omega/T}d\omega - \frac{133}{9\times 2^{10}}\int_0^{\omega_{c1}}\omega^4 e^{-\omega/T}d\omega\langle\frac{\alpha_s}{\pi}GG\rangle \\ &= \mathcal{F}(\omega_{c1},T). \end{aligned} \tag{8}$$

The binding energy $\bar{\Lambda}$ of the F-wave meson states can be estimated from Eq.(8) by taking the derivative about $(-\frac{1}{T})$ on both sides, that is,

$$\bar{\Lambda}_{+,5/2} = \frac{1}{2\mathcal{F}(\omega_{c1},T)}\frac{d\mathcal{F}(\omega_{c1},T)}{d(-\frac{1}{T})}. \tag{9}$$

One can also estimate the universal form factor $\tau(y)$ via three-point function sum rules by considering the three-point correlator

$$i^2\int d^4x d^4z e^{i(k^{'}\cdot x - k\cdot z)}\langle 0|T[J^{\alpha\beta}_{2,+}(x)J^{\mu(v,v^{'})}_{V,A}(0)J^{\dagger}_{0,-}(z)|0\rangle = \Gamma(\omega,\omega^{'},y)\mathcal{L}^{\mu\alpha\beta}_{V,A}, \tag{10}$$

where $J^{\mu(v,v^{'})}_V = h(v^{'})\gamma^\mu h(v)$ and $J^{\mu(v,v^{'})}_A = h(v^{'})\gamma^\mu\gamma_5 h(v)$. The variables $k(=P-m_b v)$ and $k^{'}(=P'-m_c v')$ denote residual "off-shell" momenta of the initial and final meson states, respectively. For heavy quarks in bound states they are typically of order $\Lambda_{QCD}$ and remain finite in the heavy quark limit. $\Gamma(\omega,\omega^{'},y)$ is an analytic function in the "off-shell" energies $\omega = 2v\cdot k$ and $\omega' = 2v'\cdot k'$ with discontinuities for positive values of these variables. It also depends on the velocity transfer $y = v\cdot v'$, which is fixed in a physical region. $\mathcal{L}_{V,A}$ is the Lorentz structure. Following the standard process of the QCD sum rule method and confining us to the operators of dimension $D\leq 5$ in OPE, the resulting equation reads

$$\begin{aligned} \tau(y)f_{-,1/2}f_{+,5/2}e^{-(\bar{\Lambda}_{-,1/2}+\bar{\Lambda}_{+,5/2})/T} &= \int_0^{\omega_{c0}}\int_0^{\omega_{c1}}d\nu d\nu^{'}e^{-\frac{\nu+\nu^{'}}{2T}}\rho_{pert}(\nu,\nu^{'},y) \\ &- \frac{T^2}{3\times 2^4}\frac{4y+7}{(y+1)^3}\langle\frac{\alpha_s}{\pi}GG\rangle. \end{aligned} \tag{11}$$

We have performed a double Borel transformation in $\omega$ and $\omega^{'}$ on both sides of the sum rule and take the Borel parameters to be equal [12, 13]: $T_1 = T_2 = 2T$. The perturbative

spectral density is

$$\rho_{pert}(\nu,\nu^{'},y)=\frac{3}{2^9\pi^2}\frac{1}{(y+1)^{\frac{5}{2}}(y-1)^{\frac{7}{2}}}\nu^{'}[(5\nu-12y\nu^{'}-3\nu^{'})\nu^2+(3\nu-\nu^{'})(2y^2+2y+1)\nu^{'2}]$$
$$\times\Theta(\nu)\Theta(\nu^{'})\Theta(2y\nu\nu^{'}-\nu^2-\nu^{'2}). \quad (12)$$

Following the discussions in Refs. [12, 18], we must make a change for the integral variables: $\nu_-=\nu-\nu^{'}$, $\nu_+=\frac{\nu+\nu^{'}}{2}$ and choose the triangular region defined by the bounds: $0\le\nu_+\le\omega_c$, $-2\sqrt{\frac{y-1}{y+1}}\nu_+\le\nu_-\le2\sqrt{\frac{y-1}{y+1}}\nu_+$. As discussed in Refs. [12, 18], the upper limit $\omega_c$ for $\nu_+$ in the region $\frac{1}{2}[(y+1)-\sqrt{y^2-1}]\omega_{c0}\leqslant\omega_c\leqslant\frac{1}{2}(\omega_{c0}+\omega_{c1})$ is reasonable. After the integral over the "off-diagonal" variable $\nu_-$ has been done, the final sum rule for $\tau$ appears to be

$$\tau(y)f_{-,1/2}f_{+,5/2}e^{-(\bar\Lambda_{-,1/2}+\bar\Lambda_{+,5/2})/T} = \frac{3}{5\times2^4\pi^2}\frac{1}{(1+y)^4}\int_0^{\omega_c}d\nu_+e^{-\frac{\nu_+}{T}}\nu_+^5 - \frac{T^2}{3\times2^4}\frac{4y+7}{(y+1)^3}\langle\frac{\alpha_s}{\pi}GG\rangle. \quad (13)$$

Numerical calculations are straightforward. We first estimate the mass of the final charmed doublet from the two-point sum rule (9). The stability window exists when $\omega_{c1}$ lies in the interval 3.5 to 3.7 GeV. When $\tau(y)$ is estimated, the systematic uncertainties can be reduced through dividing the three-point sum rule (13) by the square roots of two-point sum rules (7) and (8), as many authors did [12, 13]. Imposing the usual criteria for the upper and lower bounds of the Borel parameter, we found that they have a common sum rule "window": 0.7GeV $<T<$ 1.5GeV, which overlaps with that of the two-point sum rule (7)[see Fig. 1(a)]. Notice that the Borel parameter in the sum rule for the three-point correlator is twice the Borel parameter in the sum rule for the two-point correlator. In the evaluation we have taken 2.0GeV $<\omega_{c0}<$ 2.4GeV, which is fixed by analyzing the corresponding two-point sum rule [12, 13]. According to the discussion above, we can fix $\omega_c$ in the region 2.8GeV $<\omega_c<$ 3.0GeV. For the QCD parameters entering the theoretical expressions, we take the standard values: $\langle\overline{q}q\rangle=-(0.24)^3\text{GeV}^3$, $\langle\alpha_sGG\rangle=0.04\text{GeV}^4$, and $m_0^2=0.8\text{GeV}^2$. Taking all these into account, the bounding energy of the final meson doublet is found to be

$$\bar\Lambda_{+,5/2}=1.58\pm0.08\text{GeV}. \quad (14)$$

The evaluation of $\tau(y)$ is shown in Fig. 1(b). The resulting curve can be parametrized by the linear approximation

$$\tau(y)=\tau(1)[1-\rho_\tau^2(y-1)],\ \tau(1)=0.1\pm0.02,\ \rho_\tau^2=0.15\pm0.02. \quad (15)$$

The errors mainly come from the uncertainty due to $\omega_c$ and $T$. It is difficult to estimate these systematic errors which are brought in by the quark-hadron duality.

The differential decay rates are calculated by making use of formulas (1) and (2) given above:

$$\frac{d\Gamma}{dy}(B\to D_2^{*'}\ell\overline{\nu})=\frac{G_F^2V_{cb}^2m_B^2m_{D_2^{*'}}^3}{360\pi^3}(\tau(y))^2(y+1)^{\frac{5}{2}}(y-1)^{\frac{7}{2}}[(1+r_1^2)(7y+3)-2r_1(4y^2+3y+3)], \quad (16)$$

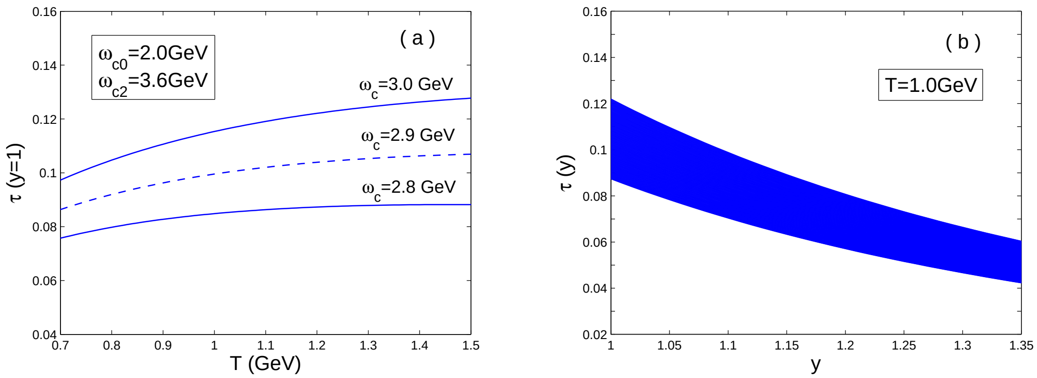


FIG. 1: (a) *Dependence of* $\tau(y)$ *on the Borel parameter* $T$ *at* $y = 1$.*(b) Prediction for the Isgur-Wise function* $\tau(y)$.

$$\frac{d\Gamma}{dy}(B \to D_3\ell\overline{\nu}) = \frac{G_F^2 V_{cb}^2 m_B^2 m_{D_3}^3}{360\pi^3}(\tau(y))^2(y+1)^{\frac{5}{2}}(y-1)^{\frac{7}{2}}[(1+r_2^2)(11y-3)-2r_2(8y^2-3y+3)], \tag{17}$$

with $r_i = \frac{m_{D_i^{**}}}{m_B}$ ($D_i^{**} = D_2^{*'}, D_3$ for $i = 1, 2$ ). Taking into account that the c-quark mass is $m_c = 1.27$GeV, the mass of the $s_l^P = \frac{5}{2}^+$ charmed doublet appears to be 2.85GeV. For the mass of the initial $B$ meson, we take 5.279GeV[19]. The maximal values of $y$ are then $y_{max}^{D_2^{*'}} = y_{max}^{D_3} = (1 + r_{1,2}^2)/2r_{1,2} \approx 1.196$. By using the parameters $V_{cb} = 0.04$, $G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$, the semileptonic decay rates of $B \to (D_2^{*'}, D_3)\ell\overline{\nu}$ turn out to be $1.85 \times 10^{-19}$GeV and $1.78 \times 10^{-19}$GeV, respectively. Considering that $\tau_B = 1.638$ps [19], the branching ratios are $4.6 \times 10^{-7}$ and $4.4 \times 10^{-7}$, respectively. So we can see that the contributions of the decay modes $B \to (D_2^{*'}, D_3)\ell\overline{\nu}$ to the inclusive semileptonic $B$ decay rate are negligibly small. The severe suppression is mainly due to the orthogonality of the F-wave orbitally excited spatial wave function of $D_i^{**}$'s to B's. As expected, the dynamical suppression in these processes is much severer than the CKM suppression in the $b \to u\ell\nu$ processes.

In summary, we have estimated the leading-order universal Isgur-Wise function describing the $B$ meson of ground-state transition into orbitally excited $F$-wave charmed resonances, the $(2^+, 3^+)$ states $(D_2^{*'}, D_3)$, which belong to the $s_l^P = \frac{5}{2}^+$ heavy quark doublet, by use of the QCD sum rule within the framework of HQET. The semileptonic decay widths we predict are $1.85\times10^{-19}$GeV and $1.78\times10^{-19}$GeV. The corresponding branching ratios are $4.6\times10^{-7}$ and $4.4\times10^{-7}$, respectively. They are much smaller than those of the $B$ to low-lying charmed meson modes and even the CKM suppressed modes. Therefore, their contributions to the inclusive semileptonic $B$ decay rate are nearly negligible. We cannot expect the high-excited charmed semileptonic modes to contribute much to the total $B$ decay in the experiments because of the severe dynamical suppression.

**Acknowledgments**

This work was supported in part by the National Natural Science Foundation of China under Contract No. 10675167.

---

[1] M. Neubert, Phys. Rep. 245, 259 (1994) and references therein.

[2] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

[3] Y. B. Dai, C. S. Huang, M. Q. Huang, and C. Liu, Phys. Lett. B 390, 350 (1997); Y. B. Dai, C. S. Huang, and M. Q. Huang, Phys. Rev. D 55, 5719 (1997).

[4] Y. B. Dai, C. S. Huang, M. Q. Huang, H. Y. Jin, and C. Liu, Phys. Rev. D 58, 094032 (1998).

[5] X. H. Zhong and Q. Zhao, Phys. Rev. D 78, 014029 (2008).

[6] W. Wei, X. Liu, and S. L. Zhu, Phys. Rev. D 75, 014013 (2007).

[7] N. Isgur and M. B. Wise, Phys. Rev. D 43, 819 (1991).

[8] A. K. Leibovich, Z. Ligeti, I. W. Stewart, and M. B. Wise, Phys. Rev. Lett.78, 3995 (1997); Phys. Rev. D 57, 308 (1998).

[9] A. Anastassov et al. (CLEO Collaboration), Phys. Rev. Lett. 80, 4127 (1998).

[10] D. Buskulic et al. (ALEPH Collaboration), Phys. Lett. B 395, 373 (1997); Z. Phys. C 73, 601 (1997).

[11] B. Aubert et al. (BARBAR Collaboration), hep-ex/0808.0333.

[12] M. Neubert, Phys. Rev. D 45, 2451 (1992); ibid. 46, 3914 (1992).

[13] M. Q. Huang and Y. B. Dai, Phys. Rev. D 59, 034018 (1999); ibid. 64, 014034 (2001); M. Q. Huang, C. Z. Li, and Y. B. Dai, Phys. Rev. D 61, 054010 (2000).

[14] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D 61, 014016 (1999); Phys. Rev. D 75, 074008 (2007); hep-ph/0006187.

[15] L. F. Gan and M. Q. Huang, Phys. Rev. D 79, 034025 (2009).

[16] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. 343,1 (1990); A. F. Falk, Nul. Phys. B 378, 79 (1992).

[17] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); ibid. 147, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. 32, 11 (1984).

[18] B. Blok and M. Shifman, Phys. Rev. D 47, 2949 (1993).

[19] Particle Data Group, C. Amsler et al., Phys. Lett. B 667, 1 (2008).